Mathematizing Phenomenology*

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Husserl is well known for his critique of the "mathematizing tendencies" of modern science and is particularly emphatic that mathematics and phenomenology are distinct and in some sense incompatible. But Husserl himself uses mathematical methods in phenomenology. In the first half of the paper, I give a detailed analysis of this tension, showing how those Husserlian doctrines which seem to speak against application of mathematics to phenomenology do not in fact do so. In the second half of the paper I focus on a particular example of Husserl's "mathematized phenomenology": his use of concepts from what is today called dynamical systems theory.

Key Words: Edmund Husserl, Mathematization, Dynamical Systems Theory, Formalization, Naturalism.

In light of Husserl's extensive analyses of the structure of consciousness, it has been hoped that his phenomenological program will have insights to contribute to cognitive science.[1] But Husserl's vigorous critique of naturalism and its "mathematizing tendencies" seems to argue against such interaction. In particular, Husserl seems to suggest that any application of mathematical concepts to phenomenology would amount to a category error, since phenomenological results should only be based on pure intuition. In *Ideas 1*, for example, Husserl says:

> …the inquiry of phenomenology into Pure Consciousness sets itself and needs set itself no other task than that of making such descriptive analyses as can be resolved into pure intuition, [thus,] the theoretical framework of the mathematical disciplines and all the theorems which develop within it cannot be of any service (p. 160).

In what follows I argue that one would be mistaken to interpret these and related passages as prohibitions against the use of mathematical techniques in phenomenological

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[1] See, e.g., McIntyre (1986), Mensch (1991), Chokr (1992), Sharoff (1995), Van Gelder (1996), Petitot et. al (1999), and the contributions to this journal, which was launched in 2002 to provide a forum for studying intersections between phenomenology and the cognitive sciences.

setting. Husserl's critique targets various naïve uses of mathematics and emphasizes differences between phenomenology and mathematics. However, Husserl allows that mathematical concepts and techniques can be applied to phenomenological problems, so long as they are properly understood and phenomenologically grounded. In fact, on Husserl's own principles we *ought* to use mathematics, in phenomenology as in any complex domain of inquiry, insofar as symbolic methods allow us to transcend the "limits…imposed upon" us by the "finitude of human nature" (*PA*, p. 202).

In the first half of the paper I reconcile Husserl's critique of mathematics with his own deployment of mathematical techniques. I begin by showing how this work relates to other studies of Husserl and the formal sciences. I then present evidence that Husserl used mathematical techniques in phenomenological settings. Finally, I formulate Husserl's critique of mathematics and resolve the apparent tensions that result.

In the second half of the paper I focus on a particular case of "mathematized phenomenology," Husserl's use of basic concepts of dynamical systems theory. I focus on dynamical systems theory because it is prevalent in contemporary cognitive science and because it is here that I see the most potential for interaction between (mathematized) phenomenology and the cognitive sciences. Moreover, dynamical systems theory is already drawn on by neurophenomenologists.[2] However, there has not yet been a systematic analysis of the basic concepts of dynamical systems theory relative to phenomenology. We shall see that Husserlian phenomenology and dynamical systems theory are both founded on the basic principle that systems can be understood in terms of their possibilities. One finds in each case a space of possibilities, which is multi-

[2] See Petitot et. al. (1999); especially the contributions by Petitot, Van Gelder, and Varela.

dimensional, has topological and geometric structure, and whose members must be instantiated in accordance with rules in order for coherent behavior to arise.

Background

An extensive literature has developed around Husserl's contributions to the philosophy of logic and mathematics.[3] Husserl thought that logical and mathematical truths should be grounded on the "primitive soil" (*urboden*) of authentic insight. Such towering 20th century figures as Hermann Weyl and Kurt Gödel were influenced by this strand of Husserl's thinking. Gödel sought a way to provide phenomenological foundations for mathematics in light of, among other things, his own incompleteness theorems.[4] Weyl sought to connect developments in 20th century physics with the authentic intuition of space, and also to connect his foundational views on mathematics with Husserl's phenomenology.[5] Husserl's student Becker attempted to provide phenomenological foundations for Cantor's transfinite arithmetic, via the iterated operations of reflection Husserl described.[6]

But grounding logic and mathematics in phenomenology is different than applying mathematics to phenomenology, and it is the latter that I focus on. As we shall see, once mathematical disciplines have been authenticated by phenomenological methods, Husserl thought they could be fruitfully reflected back on phenomenology itself.

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[3] See, e.g., Tragesser (1977, 1984), Willard (1984), Tieszen (1989, 2005). Hill and Haddock (2000), and the various contributions to a special issue of *Philosophia Mathematica* on "Phenomenology and Mathematics" (volume 10, 2002).
[4] See Tieszen (2005), ch. 6.
[5] See Mancosu and Ryckman (2002).
[6] See Mancosu and Ryckman (2002).

This project has precedent. Perhaps the most developed area of work formalizing and in some sense mathematizing phenomenological themes is in the area of mereology, the study of parts and wholes.[7] There has also been work formalizing Husserl's theory of time consciousness,[8] his method of free variation,[9] his theory of intentionality and perception,[10] and his horizon theory.[11] This paper has a two-fold purpose relative to these projects: (1) to validate them relative to Husserl's own critique of mathematics, and (2) to supplement them with an explicit discussion of dynamical systems theory.

In the next section I focus on prominent examples of application of mathematics to phenomenology; I defer most of the analysis of Husserl's incipient dynamical systems theory to the final section.

Mathematical Tendencies in Husserlian Phenomenology

Husserl began his career as a mathematician working on the calculus of variations under Weierstrass in Berlin. This early experience with mathematics, especially as personified by Weierstrass, made a stark contrast with philosophy, which Husserl initially found sloppy and disappointing. As he put it in a late, retrospective letter (written at the age of 60):

> Thanks to Weierstrass and his thoroughgoing mathematics, I was used to an intellectual neatness, but I found that contemporary philosophy, which makes so much of its scientific approach, in fact falls far short of it (*SW*, p. 360).

[7] See Smith B. (1982), Blecksmith and Null (1992), Fine (1995).
[8] See Miller (1984).
[9] See Tieszen (2005), ch. 3.
[10] See Smith and McIntyre (1982), Miller (1984), Baruss (1989), Krysztofiak (1995), and Petitot (1999).
[11] See Smith and McIntyre (1982) and Krysztofiak (1995).

Indeed, after completing his dissertation in mathematics in 1882, Husserl spent several decades fully engaged with the formal sciences. Hundreds of pages of manuscript from this period show Husserl wrestling with problems in logic and mathematics.[12] Proofs, geometric diagrams, and formulae are relatively abundant in these early texts.

Husserl's application of symbolic techniques to phenomenology begins in the same period. In the *Logical Investigations* (written in the 1890's and first published in 1900), for example, Husserl describes a "presentation operation" which can be "iterated" in a sequence "O, $P(O)$, $P(P(O))$…" (*LI*, p. 643). "O" corresponds to an intentional object, for example a dog. "$P(O)$" corresponds to a direct, first-order presentation of an intentional object, in this case a perception of a dog. "$P(P(O))$" corresponds to a presentation of that presentation, in this case, an imagination of that dog. Husserl's point is that we can iterate the presentation operator indefinitely many times. We can present the presentation of the dog, we can present that presentation, etc. That is, we can imagine ourselves imagining the dog, we can imagine that imagination, and so on (also see *PP*, p. 156).[13]

In the sixth *Logical Investigation* Husserl introduces a "symbolic equation" which describes the relative weights of intuitive and conceptual components in perceptual acts. (*LI* , p. 732). The equation is "$i + s = 1$", where "i" corresponds to the intuitive weight of a perceptual act and "s" corresponds to its signitive or conceptual weight. Although the equation itself (like many of Husserl's tentative formalizations) is flawed,[14] the strategy it

[12] See *Husserliana* volume 21, *Aufsätze und Rezensionen*, and volume 22, *Studien zur Arithmetik und Geometrie*.
[13] Though Husserl does not put it this way, P is a map on the set of possible intentional states which can be used to partition it into the infinite sequence of equivalence classes "perception, first-order imagination, second-order imagination, etc." Note that the set of intentional acts is closed under P, no result of the presentation operator will ever fall outside the class of intentional states.
[14] The equation can be interpreted in two ways, both of which are problematic. On one, more natural interpretation, i and s represent separately varying phenomena which constrain one another. This interpretation implies that as the

embodies is significant, for it shows that Husserl was willing to associate mental states with numbers in order to reason about them via their numerical representations.[15] In particular, Husserl assumes that perceptual acts can be associated with numbers in the interval [0, 1] which symbolize their intuitive and conceptual weightings. For example, a perceptual act associated with an intuitive content of 0 and a conceptual content of 1 is a "pure signitive presentation" devoid of sensory content, e.g. a mere thought of a dog or a house. Husserl continues this line of thought in section 24, where he argues that one can speak meaningfully of "distances" between perceptual acts relative to their greater or lesser "perfection" (which he cashes out in terms of "differences of greater or lesser completeness, liveliness and reality"). For example, as one approaches a tree which was initially an indistinct object in the distance, the relevant perceptual acts are associated with greater and greater degrees of perfection, B_1, B_2, ….[16] Husserl assumes that the "greater than" relation is transitive: "If B_2 is at once $> B_1$, and $B_3 > B_2$, then $B_3 > B_1$, and this last distance exceeds those which mediate it" (*LI*, p. 735).[17]

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intuitive weighting of a perceptual act increases its conceptual weighting diminishes (Compare any equation in which the sum of two quantities equals a positive constant; e.g. an equation which says that total number of predator and prey equals 100, so that as more predators enter a population there are less prey, and vice-versa.) But this is problematic, insofar as one very often learns more about an object as he or she sees more of it, which suggests that both the intuitive and conceptual components of an act can simultaneously increase. On another interpretation, the equation is simply describing a gradient from the case where $i = 0$ to the case where $i = 1$. Compare an equation which says that a square can be all white, all black, or some level of gray (white + black = 1). In that case we only have a single phenomenon which varies—let us say, amount of black coloring. The other variable, which represents absence of the first property, "tags along," as it were, insofar as it does not represent an independently varying phenomenon. However, one would not normally introduce an "equation" to describe a simple gradient, and doing so in this case is misleading, for it implies the first interpretation.

[15] The study of numerical representations and their properties is an area of research which began during Husserl's lifetime (with Von Helmholtz) and continues today. See Narens (2002). On this type of analysis one would explicitly introduce a function from perceptual acts to numbers, which Husserl does not do (though, as we shall see, he was very much aware of the ontological distinction between real phenomena and their symbolic representation). As a result, he is sometimes not clear whether he is speaking of relations between actual mental states or between their symbolic representations.

[16] Husserl is not clear whether the B_i name the perceptual acts themselves, their degree of perfection, their "image" components, or some other "inner moment" of the acts.

[17] Husserl bases the ordering relation on distances between perceptual acts. This assumes that the distances are more fundamental than the ordering relation. However, this need not be the case. In fact one often finds ordering relations without distances, as in the case of preference orderings. Distances are only meaningful in interval and ratio scales, which are stronger than ordinal scales (Narens, 2002).

In *Thing and Space* (lectures of 1907) Husserl describes the structure of the visual field and the experienced-body, and how their coordinated activities constitute our sense of a three-dimensional spatial world. He develops his account "level by level," beginning with eye-movements (which expand our visual field by a fixed amount) and body movements. The "cyclical manifold" of turning, for example, is such that a rotation of the body causes the same sequence of visual experiences to periodically recur. The lectures culminate in a discussion of the constitution of changing objects, which involves a "lawfully determinate" (*TS*, p. 231) deviation from the case of unchanging objects. Consider the following diagram from the end of the lectures:

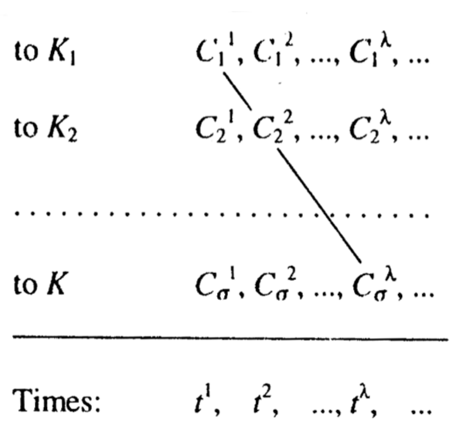
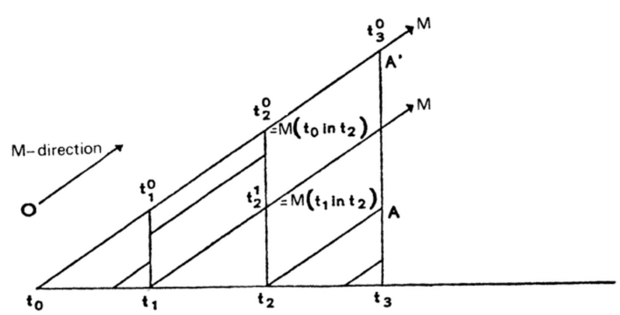


Figure 1: The constitution of objective change. From *Thing and Space*.

In the case represented in Figure 1, an object, colored differently on different sides, is changing color over time (really what is changing is a spatially extended pattern of coloration, but I will follow Husserl in speaking only of changing colors). $K_i \ldots K_\sigma$, represent different spatial perspectives on the object—the object seen from the left, the right, the back, the front, etc. $t^1 \ldots t^\lambda$ represent times. Colors—the components of the matrix—designate the perceived color of the object from perspective *i* at time *j*.[18] Rows represents the changing color of the object from a fixed perspective or "kinesthetic situation" K_i. If one were to sit still watching, say, a glowing fire from time 1…λ., this would correspond to one row of the diagram. If one were to walk around the fire, observing changing colors from changing perspectives, this would correspond to a curve through the diagram, e.g. the diagonal Husserl marks with a solid line. Indefinitely many "trains of perception" are possible in this "bidimensional manifold of perception" (*TS*, p. 231), and each such train corresponds to a sequence of components in the matrix above (columns do not correspond to possible perceptual processes, but rather to possible colors relative to possible perspectives at a fixed point in time).

In the time manuscripts of 1895-1917, Husserl again describes symbolic laws (*Time*, pp. 44-45) and iterated operations (*Time*, p. 215) which apply to phenomenological structures, as well as introducing his famous diagram representing two "dimensions" of time-consciousness.[19] Points on the diagram represent constituents of experiences at times.



[18] An infelicitous choice of notation, for it does not label colors themselves (which may recur, after all) and worse, makes it seem as if the colors in a given row are the same (because of the identical subscript)—which is in fact the contrast case to the one Husserl here represents.
[19] For formal treatment of Husserl's theory of time-consciousness see Miller (1984).

Figure 2: The memorial transformations of time-consciousness, from the lectures of 1917.

The horizontal axis of this diagram represents objective time as well as the experienced flow of time. Points on the horizontal correspond to "primary experiences" of things in the immediate present. Vertical lines represent total experiences (primary experiences together with memories and anticipations) at times, and the sections of vertical lines above the horizontal axis represent parts of an experience corresponding to our short-term memories or "retentions" of objects as they were in the immediate past. The diagonals represent the memorial transformations primary experiences go through in the passage of time. Suppose at t_1 I hear the word "good" said out loud. At t_2 I hear "dog." At the moment I hear "dog" said out loud I have a retentional memory of the word "good" having been said a moment ago (hence my ability to hear this as "good dog"). The memorial transformation of "good" as a primary experience at t_1 to a "primary memory" or "retention" at t_2 is represented in the diagram as $t_2^1 = M(t_1 \text{ in } t_2)$.

Although explicit mathematization drops off in the 20's (when Husserl was in his sixties), Husserl was concerned with issues concerning symbolic techniques vis-à-vis phenomenology to the end of his life, as evidenced by his three late studies of logic: *Analyses of Passive and Active Synthesis: Lectures on Transcendental Logic* (1920-1925), *Formal and Transcendental Logic* (1929), and his posthumously published *Experience and Judgment*. The overarching theme of these works—logic and its origins in conscious life—is treated of in largely qualitative terms, though in the *Analyses of Passive and Active Synthesis* Husserl does make occasional use of formal apparatus. In section 31, for example, Husserl outlines some "Problems of a Phenomenology of Sense

Fields," making explicit reference to "mono-dimensional manifolds" (e.g. the set of points comprising a perceived line), the "velocity" of color-change in the visual field (which must rapidly increase and decrease for a border to be perceived),[20] and "axioms" of sense perception. For example, Husserl takes it as axiomatic that line-segments in the visual field are comparable according to length. He also takes it as axiomatic (wrongly, I think), that vicinity relations in the visual field are transitive "If *a* is in direct vicinity to *b*, and *b* is in direct vicinity to *c*, then *a* is in direct vicinity to *c*" (*APS*, p. 195).[21]

Husserl's Critique of Mathematization

Husserl's mathematical approach to phenomenology is easily overlooked in light of his many critical comments about mathematics and his stated opposition to its use in (pure) phenomenology.[22] In *Ideas 1*, for example, he says "the theoretical framework of the mathematical disciplines and all the theorems which develop within it cannot be of any service [to phenomenology]" (*Ideas,* p. 160). Elsewhere in *Ideas 1* he claims that the "precise aims and methods [of mathematical disciplines]… should in principle be unsuited for the sphere of experience" (*Ideas 1,* p. 185). In his late works, Husserl's indictments of mathematical natural science are especially vigorous, verging on ridicule. In one passage he suggests that mathematical scientists, faced with the crises of the times, "find formulas with which to console themselves and their readers" (*Crisis*, p. 11).

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[20] References to velocity and acceleration of perceptual change are made throughout the corpus. For example, in the example from *Thing and Space* above, where the color of a thing on each side is changing, Husserl explicitly points out that this change has velocity and acceleration (*TS*, p. 230).
[21] Other examples of mathematization occur in *Time* p. 243, and *Thing and Space* pp. 153, 165, 168, 176, 230, 267. See also *Die `Bernauer Manuskripte' über das Zeitbewußtsein*.
[22] For comprehensive discussion of Husserl's theory and critique of science, including references to the earlier literature, see Tieszen (2005), especially chapter 1.

In light of Husserl's own deployment of mathematical methods in phenomenology, we seem to face a paradox: Husserl simultaneously opposed and made use of mathematical techniques in phenomenology. In this section I resolve the apparent paradox.

Husserl makes at least six claims which seem to oppose the use of mathematics in phenomenological contexts:

> 1. "Mathematization" is problematic in natural science and is a root of the crisis of Western man.
> 2. Mathematical methods are prohibited from pure phenomenology.
> 3. Mathematics is a formal discipline, phenomenology is material discipline.
> 4. Mathematics is an exact discipline, phenomenology is a morphological discipline.
> 5. Mathematics is deductive, phenomenology is not.
> 6. Complete mathematical theories are satisfied by "definite manifolds", phenomenological theories are not.

None of 1-6 speaks against application of mathematical methods to phenomenological problems, though the passages in which 1-6 are expressed do, on a cursory read, seem to suggest a hostility on Husserl's part to mathematical methods, in particular as applied to phenomenology. The resolutions of the apparent conflicts are as follows. (1) targets an extreme form of mathematization which confuses real objects with mathematical objects. But use of mathematical objects to *represent* or *symbolize* real processes does not make this mistake and is in fact essential to certain kinds of inquiry, as Husserl explicitly says in *Philosophy of Arithmetic*. (2) concerns phenomenology as a "pure" foundational enterprise; it targets the use of mathematical and logical concepts insofar as they are dogmatically assumed to be true independently of phenomenology; however, we can make use of mathematical methods once they have been phenomenologically justified.

(3-6) simply outline differences between phenomenology and mathematics as disciplines and say nothing against the application of the latter to the former.

Though none of 1-6 speak against application of mathematics to phenomenology, each does highlight a potential confusion, a danger associated with a naïve, uncritical, or overly enthusiastic application of mathematical methods. Thus, in resolving the apparent tensions 1-6 generate, we see how mathematics can be safely applied to phenomenological problems, or indeed to problems in any material domain (biology, chemistry, social science, etc.). What emerges from this section, then, is a theory of the proper scope and conduct of applied mathematics.

Let us consider each of 1-6 in more detail.

> 1. "Mathematization" is problematic in natural science and is a root of the crisis of Western man.

Husserl's last works, in particular *The Crisis of European Sciences and Transcendental Phenomenology*, were written with uncharacteristic vigor, attuned as Husserl was to the crises being felt throughout Europe in the mid 1930's.[23] Husserl's central contention in these late works is that the crises of the times arose, at least in part, from a naïve and often unexpressed belief in objectivism (also "naturalism")[24], the idea that true being is physical being, and that physical being is best described mathematically. In its worst and most influential incarnation, objectivism confuses the mathematics which describes the

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[23] Husserl faced personal and professional crises in this era. He lost a son and a close student (Adolff Reinach) in the Great War and faced persecution by the Nazis in his last years. Husserl also faced a crisis in his own work—he was coming out of a decades-long period of public silence and personal insecurity, during which he had seen his philosophy eclipsed by such diverse trends as existentialism, psychoanalysis, behaviorism, and naturalism (see Welton (2000), introduction and ch. 5). Moreover, his former protégé Heidegger had turned against him, at first privately, but with increasing virulence in the late 20's and 30'—events which culminated, famously, in Heidegger's denying Husserl the use of the university library at Freiburg and removing the dedication to Husserl from *Being and Time* (see *Psychological and Transcendental Phenomenology and the Confrontation with Heidegger 1927-1931*).

[24] For definitions and discussion of these terms, see (*Crisis*, pp. 68, 292).

physical universe with the universe itself. As Husserl put it, "… nature itself becomes a mathematical manifold" (*Crisis*, p. 54) that is, a set of mathematically determined physical possibilities.

Using a historical methodology similar to Heidegger's, Husserl traces the historical process by which the world of mathematical constructed "idealities" came to describe and ultimately replace the world of intuitively given nature, the "lifeworld" of everyday experience (see *Crisis*, p. 51; also see *EJ*, pp. 43-44). Husserl describes this process in terms of a "garb of ideas" which "dress" reality up in mathematical symbolism:

> Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally… *dresses it [the lifeworld] up* as "objectively actual and true" nature (*Crisis*, 51).

As mathematical constructions replace the world of everyday experience, everyday experience is correspondingly transformed and diminished into an imperfect medium of access to objectively true physical nature. Vagaries of perspective, errors of measurement, individual bias, etc. are precisely what science endeavors to eliminate. Conscious events, as conceived by science, only have value insofar as they "vaguely indicate an in-itself which lies behind this world of possible experience" (*Crisis*, p. 54).

For Husserl these transformations (of lifeworld into mathematical world and of consciousness into imperfect medium of access to that world) represent an ontological inversion, a reversal of epistemic and philosophical priorities of the worst kind, and one root of the crisis of modern man.

> [Science] excludes in principle precisely those questions which man, given over in our unhappy times to the most portentous upheavals,

> finds the most burning: questions of the meaning or meaninglessness of the whole of this human existence… The mere science of bodies clearly has nothing to say: it abstracts from everything subjective (*Crisis*, p. 6).[25]

But Husserl is here arguing against those who replace the real world with a mathematical world. None of what he says speaks against *applications* of mathematics to the real world. One need only distinguish the symbols used in mathematics from the real (physical, biological, phenomenological) processes those symbols represent. As Husserl says in *Philosophy of Arithmetic*, symbolic representations "serve us as a provisional surrogate" for actually perceived phenomena (*PA*, p. 206). In fact, symbols provide an essential cognitive heuristic, insofar as they allow us to reason about phenomena that would otherwise be inaccessible to our finite intellects. Husserl develops this point in detail in his earliest works, especially part II of the *Philosophy of Arithmetic*, where he investigates the ontology and cognitive status of symbol systems.[26] He there assumes we only have "authentic" number concepts for small numbers, e.g. four, insofar as we can directly perceive collections of four things *as* collections of four things.[27] Beyond that we must rely on symbolic or "inauthentic" representations, which allow us to transcend the "limits…imposed upon" us by the "finitude of human nature" (*PA*, p. 202). Husserl goes so far as to say, "the whole of arithmetic is nothing other than a sum of artificial devices for overcoming the essential imperfections of our intellect" (*PA*, p. 202). Husserl makes this point vividly in a footnote (*PA*, p. 202), where he says that God has no need of symbols and calculation procedures, for he has authentic intuitions of every number

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[25] These passages are late reflections of a long-standing concern on Husserl's part that ideal constructions not be confused with directly perceived entities, and that reified methodological constructions not be confused with the conscious processes which give them sense. As Dallas Willard, citing Husserl's 1891 review of Schröder, points out, the "theme of a lack of self-understanding—and even of self-deception—on the part of logic and logicians is no mere afterthought on the part of the later Husserl. Rather, it is one dealt with in detail in the early writings" (Willard 1979, pp. 144-145). Themes similar to those found in *Crisis* are also prefigured in *Ideas 3*.

[26] For discussion see Willard (1980, p. 60 and following).

[27] Roughly through the number ten (*PA*, pp. 236-237).

concept and indeed of every concept whatsoever. Thus, Gauss is wrong to say that "God arithmetizes," for God has no need for the cognitive aid arithmetic supplies. But man *does* arithmetize, and he does so of practical necessity.

A puzzle arises at this point. Husserl's distinction between authentic concepts, whose objects can be "directly given," in intuition, and symbolic or inauthentic concepts, which represent "indirectly through signs" (*PA*, 206), clearly generalizes from numbers to all of mathematics. In fact, Husserl seemed to think most of mathematics was inauthentic—detached from direct intuitions of things (at least for finite beings) but providing useful cognitive aids. But it seems as if phenomenological claims should be paradigmatically authentic. If the claims of any science correspond to direct intuitions, those of phenomenology ought to! But insofar as mathematics consists of inauthentic symbolic techniques, mathematization renders phenomenology inauthentic, and hence not subject to direct intuition.

In fact, I think this is both right and essential. It is precisely the point of applied mathematics to provide a reasoning procedure to assist us where direct intuition is limited. While direct reflection on mental acts is an important starting point for phenomenological investigation, many of Husserl's more suggestive claims involve studying relationships between structured manifolds of possible mental acts and their mereological parts. As we shall see, these manifolds correspond to continuous infinites of possible mental states. While it is possible to compare a few mental acts by imagining them together, to intuitively fulfill a claim about an infinite manifold of possible mental acts is not possible for a finite being. One cannot, for example, simultaneously see an object from every one of its sides, though one can reason symbolically about the structured manifold which represents all those possibilities. And again, that is precisely the power of applied mathematics: it provides a means of reasoning about matters that would otherwise be inaccessible to us.

Consider Husserl's time diagrams (figure 2 above). It is impossible in one mental state to reflect on every moment of a continuous stream of mental states. So the diagram is a kind of symbolic heuristic, an inauthentic representation of a conscious process. However, these diagrams allowed Husserl to identify structures he might not have otherwise noticed. Using the diagram, he was able to clearly describe the relationship between the flow of experience in subjective time (the horizontal line), the totality of retentions present in a total experience at a time (the vertical lines), and the modifications primary experiences undergo over time (the diagonal lines).

> 2. Mathematical methods are prohibited from pure phenomenology.

Husserl's most explicit prohibitions of mathematics from phenomenology occur in *Ideas*, in which Husserl introduced the phenomenological reductions. The purpose of these reductions is to focus attention on conscious phenomena by "disconnecting" or "bracketing" all non-phenomenological sources of evidence. Phenomenology is thereby purified of illicit influence (hence "pure phenomenology"). In particular, no piece of scientific or mathematical knowledge should be taken as foundational or intrinsically legitimate:

> [Even though] all sciences which relate to this natural world… fill me with wondering admiration… I disconnect them all, I make absolutely no use of their standards, I do not appropriate a single one of the propositions that enter

> into their systems…. I take none of them, no one of them serves me for a foundation (*Ideas,* p. 100).

This applies to particular empirical sciences (“no transcendent-eidetic regions and disciplines can contribute, in principle, any premises at all” (*Ideas*, p. 162)), as well as mathematical ones (“the theoretical framework of the mathematical disciplines and all the theorems which develop within it cannot be of any service” (*Ideas*, p. 160)).

However, *once phenomenologically grounded*, Husserl thinks mathematical disciplines can contribute to the study of consciousness. In fact, as we just saw, Husserl thought of symbolic representations and procedures as being essential to rational inquiry. The point of the phenomenological reductions is not to remove the bracketed material, but to protect against its unwarranted use. The reductions bracket, but do not eliminate: “whatever is phenomenologically disconnected remains still, with a certain change of signature, within the framework of phenomenology” (*Ideas*, p. 346). Or again: “the bracketed matter is not wiped off the phenomenological slate” (*Ideas*, p. 194).

The reductions act as a kind of epistemic safeguard or quarantine, which ensure that data and methods used in pure phenomenological inquiry are first justified on phenomenological grounds. Rather than naïvely accepting mathematical and logical concepts as foundations for knowledge, we first trace these concepts back to their sources in intuition, and if necessary subject them to critical revision. Having done so, we are free to make use of them. Consistently with this idea, Husserl goes to great lengths to provide phenomenological foundations for those formal and mathematical concepts he draws on within phenomenology (set, operation, axiom, etc.). Indeed, this is a project which preoccupied Husserl throughout his career.[28]

> 3. Mathematics is a formal discipline; phenomenology is material discipline.

In section 71 of *Ideas 1* Husserl explicitly and forcefully distinguishes mathematics and phenomenology as disciplines, and claims that mathematical methods are "in principle unsuited" for phenomenological investigation:

> let us…make clear to ourselves *the distinctive uniqueness of the mathematical disciplines in opposition to that of a theory of experiences*… and at the same time be clear as the precise aims and methods [of mathematical disciplines] which should in principle be unsuited for the sphere of experience (*Ideas*, p. 185).

Husserl emphasizes at least four differences between mathematics and phenomenology, which are considered in this and the following three sub-sections.

First, phenomenology is a material discipline which studies a particular kind of object (mental processes) while mathematics is a formal discipline whose concepts apply to all material domains.[29] However here, as with the other contrasts Husserl draws, Husserl is simply emphasizing a difference between mathematical and phenomenological

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[28] In the *Philosophy of Arithmetic* (1890) number-concepts and (in part II) axiom systems were at issue. In the *Logical Investigations* (1900), Husserl studies how the constructions of formal logic and language, in particular concepts, propositions, arguments, truths, proofs, and judgments, originate in processes of rational cognition. He later described the *Investigations* as involving a "turning of intuition back toward the logical lived experiences which take place in us whenever we think…to make intelligible how the forming of all those mentally produced formations takes place in the performance of this internal logical lived experiencing" (*PP*, p. 14). Similar concerns are at work in Husserl's late *Analyses of Passive and Active Synthesis*, *Formal and Transcendental Logic* and *Experience and Judgment*. In these works Husserl says, for example, that the logical categories of subject and predicate originate in perceptual experiences of things with properties. *Formal and Transcendental Logic* (part 1, chapter 1) contains a striking account of the phenomenology of logical inference. In his last work, the *Crisis* (especially section 9), Husserl outlines a detailed phenomenology of scientific practice, as we saw above.

[29] See *Ideas* sections 10 and 13, and *Formal and Transcendental Logic* sections 24-29.

methods, without arguing against the application of the latter to the former. In the case of the formal / material contrast it is precisely the value of formal disciplines that their results apply to all material domains, including phenomenology. Husserl makes explicit reference such applications when he refers, without critique, to "formalization on sciences of the type represented by psychology or phenomenology" (*FTL*, p. 101).

4. Mathematics is an exact discipline, phenomenology is a descriptive discipline

Let us consider a second contrast. While mathematics deals with "exact" concepts like numbers and perfect squares, phenomenology studies:

> …the phenomenologically particular object…in the whole wealth of its concreteness, precisely as it participates in the flow of experience….with just that distinctness or mistiness, that fluctuating clearness and intermittent obscurity (*Ideas,* p. 192).

Or, as Husserl also puts it, mathematics is an exact science which studies ideal objects, while phenomenology is a "descriptive" science which studies the "morphological" (in effect, inexact) essences of real objects. To conflate the two—to speak of mathematically "exact" experiences, for example—is *prima facie* absurd, a category error.[30]

However, in claiming that mathematical methods are "unsuited" to phenomenology Husserl is, again, simply pointing out that methodological tools appropriate in the exact disciplines like mathematics are inappropriate for use *within*

[30] However, Husserl does leave open whether one might pursue a kind of exact science of consciousness as a counterpart to descriptive phenomenology, which would describe perfect mental processes and their ideal structure "[There is at present no answer to] the pressing question of whether, besides the descriptive procedure, one might not follow—as a counterpart to *descriptive* phenomenology—an idealizing procedure which substitutes pure and strict ideals for intuited data and might even serve as the fundamental means for a mathesis of mental processes" (*Ideas*, p. 169).

morphological disciplines like biology and phenomenology. But of course exact concepts can be applied to inexact phenomena: equations help describe plant growth, even if there are no mathematically perfect plants, exact geometric forms can be used to describe inexact crystalline structures, etc. As Husserl says: "*exact* and *purely descriptive* sciences can indeed unite their efforts, but can never take each other's place" (*Ideas*, p. 191).

5. Mathematics is deductive, phenomenology is not

Husserl distinguishes phenomenology from mathematics and logic in another way: by pointing out that while logic and mathematics construct deductive theories, phenomenology does not, at least not exclusively. There are two aspects of this claim: First, "deductive theorizings are excluded from phenomenology" (*Ideas*, p. 193), and second, even assuming deduction were allowed, there is no list of axioms from which all phenomenological truisms could be derived: "the task of phenomenology… lies not in the systematic elaboration of… formal doctrines, wherein… we can deduce from primitive axiomatic basic formations the systematic possibilities of all further formations" (*Ideas*, p. 345).[31]

Here again Husserl's claims about formal disciplines like mathematics and geometry do not speak against their application to material disciplines like biology and phenomenology. His comments simply highlight differences between the two kinds of theory, and dangers that arise if they are confused with each other. He allows deductive

[31] Husserl makes these points vividly in connection with his critique of Descartes, who, on Husserl's analysis, suffered from the "prejudice" of believing that "under the name *ego cogito*, one is dealing with an apodictic 'axiom', which, in conjunction with other axioms… is to serve as the foundation for a deductively 'explanatory' world science, a 'nomological' science… similar indeed to mathematical natural science" (*CM*, p. 24).

inferences and axioms as a supplement to phenomenological inquiry but denies that phenomenology is wholly constituted by this type of theorizing.

With respect to deductive inference, Husserl's concern is that the theorist will allow the inference procedure to degenerate into a kind of "game," a reified calculation procedure whose inferential results are taken to be true independently of the observed phenomena.[32] But if the inferences secured by deductive methods are subject to independent verification checked using methods internal to a material discipline, in this case phenomenological observation or "direct seeing," then Husserl has no objection to their use. As Husserl says:

> …Mediate inferences are not exactly denied to [phenomenology]; but since all its cognitions ought to be descriptive, purely befitting the immanetal sphere, inferences, non-intuitive modes of procedure of any kind, only have the methodological function of leading us to the matters in question upon which a subsequent direct seeing of essences must make given… As long as that has not occurred, we have no phenomenological result (*Ideas*, p. 169, Kersten translation).

The cases where Husserl does draw deductive inferences in phenomenological contexts conform to this rule, insofar as Husserl independently verifies that the inferences drawn are consistent with independent phenomenological investigation.[33]

Husserl also allows for "axioms" of phenomenological theory, so long as they are themselves phenomenologically grounded. In *Crisis* section 9 he distinguishes "true" axioms which are "grasped with self-evidence" from "inauthentic" axioms which are

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[32] Husserl makes this point explicitly with reference to deductive systems, which are mere "games with symbols," unless they are considered with respect to "actual Objects of thinking" (*FTL*, p. 99). For a helpful discussion of the topic see Tieszen (2005), pp. 4-6.

[33] See (*Time* pp. 45-46), where Husserl "deduces...in conformity with law" a formula describing the ordering of experiences in memory. Husserl gives a simpler example in *Ideas*, where he says "we can, for instance, predicate in a 'blind' way that 2+1 = 1+2; we can, however, carry out the same judgment with insight. The positive fact [that 2+1 = 1+2]... is then primordially given, grasped in a primordial way" (*Ideas*, pp. 350-351).

merely "constructed formally." In *Ideas* section 16 Husserl refers to the former as "fundamental truths" which either apply to all regions of being or to a particular region of being. When these truths apply to all regions of being they are members of a "formal system of axioms" (e.g. the properties of sets and manifolds; on the grounding of such axioms, see footnote 28); when these truths apply to a particular region of being they are "regional axioms" (e.g. the axioms for the phenomenology of sense fields introduced in *Analyses of Passive and Active Synthesis*, section 31), which Husserl compares to Kant's synthetic *a priori* truths.

> 6. Complete mathematical theories are satisfied by "definite manifolds", phenomenological theories are not.

(6) is similar to (5) above—phenomenology is not a deductive discipline—but made from what we would today call a model theoretic standpoint (model theory studies the relationship between languages and the objects they describe). In *Formal and Transcendental Logic*, Husserl posits an "intimate" relation between theories and the objects described by theories, holding that "in all formal distinctions pertaining to judgments, differences among object-forms are included" (*FTL*, p. 79). Accordingly, the sets of objects or "manifolds" which satisfy different kinds of theories should themselves differ in kind. [34] In particular, says Husserl, the manifolds of objects which satisfy complete deductive theories are "definite," while the manifolds which satisfy non-mathematical (e.g. phenomenological) theories are not. As Husserl says, sciences like

[34] Husserlian manifolds are commonly taken to be models in the contemporary sense, though there are some differences; see (Smith 2002). Models are ordered pairs consisting of a domain of objects and an interpretation function which maps from the language to the domain. Husserl's manifolds correspond to the domain; he does not, to my knowledge explicitly introduce an interpretation function. Moreover, Husserlian manifolds correspond to intended interpretations of a theory, whereas set theoretic manifolds can consist of any arbitrary set of objects which satisfy the corresponding theory.

psychology and phenomenology lack "the systematic form of a definite deductive theory; correlatively, their provinces [models] are not definite manifolds" (*FTL*, p. 102).

A definite manifold is one which can be described by a complete deductive theory with finitely many axioms.[35] Anything which can truly be said of an object in a definite manifold can be derived from the axioms of that theory. As Husserl says: "[with a definite manifold] a finite number of concepts and propositions…determines completely and unambiguously…the totality of all possible formations in the domain, so that in principle, therefore, nothing further remains open within it" (*Ideas*, p. 187). Alternatively, one can say that any statement formed from the concepts and propositions describing a definite manifold is either derivable from the axioms of the relevant theory or is refutable by those axioms.[36]

Geometry, for example, is a "definite discipline" which governs a definite manifold of geometric objects, insofar as any true statement about geometric objects is a consequence of the axioms of geometry. Phenomenology is *not* a definite discipline, insofar as its domain of objects (possible experiences) cannot be completely delimited by a finite set of axioms:

> [Geometry] constructs a deductive theory, a theory which traces the infinity of deductive cognitions back to, or deduces them from, a small number of immediately evident truisms, its "axioms." The psychic province, however, is of a completely different essential type; it has a multiplicity of immediate essential insights which continually grows with analysis and is never to be limited (*PP*, pp. 36-37).

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[35] Cf. Tieszen (2005), p. 86. On Husserl's concept of completeness see Majer (1997) and Da Silva (2000).

[36] As Husserl says, "The axiom system formally defining such a manifold is distinguished by the circumstance that any proposition… that can be constructed…out of the concepts…occurring in that system, is either… an analytic consequence of the axioms… or an analytic contradiction" (*FTL*, p. 96). We thus have the contemporary concept of a complete theory. It is worth emphasizing that not all mathematical theories are complete. Husserl himself realized this, taking definite manifolds to be an especially strong forms of manifold and the theories which describe them ("nomologies") to be especially strong forms of theory.

Or again: "The great mathematical sciences that give natural science the *a priori* of its sphere of being arise in pure deduction out of a few axiomatic bases. It is quite otherwise in rational phenomenology. The field of immediate insights is an endless one." (*I3*, p. 39).

The important difference Husserl is marking out here is an epistemic one. All possible discoveries about objects in a definite manifold can be made by drawing deductive consequences from a finite set of axioms. "Discovery" is achieved by clever application of the rules of inference. But with a non-definite domain of objects, like the set of possible plants or the set of possible experiences, discovery requires analysis of actual cases. Drawing deductive consequences from botanical axioms is not sufficient to secure all possible botanical truths; drawing deductive consequences from phenomenological axioms is not sufficient to secure all phenomenological truths.

Here again there is no conflict with the *application* of mathematical methods to phenomenological problems. Complete theories satisfied by what Husserl called definite manifolds tend to be mathematical theories, and as we have seen, mathematical objects can safely be used to study material domains like phenomenology, as long as the two domains are kept distinct. For example, insofar as Husserl uses Euclidean geometry (a complete theory) to describe the structure of the visual field, he is using objects in a definite manifold (geometric figures) to describe objects in a non-definite manifold (visual experiences).[37] He thereby uses a system of objects which can be studied completely with respect to its theoretical description to facilitate the analysis of a system of objects which must be studied directly by the method of phenomenological intuition.

[37] Husserl thought of Euclidean geometry as the *a priori* science of "intuited world space," *Formal and Transcendental Logic*, section 29; also see *Analyses of Passive and Active Synthesis*, section 31.

To sum up points 5 and 6, Husserl believes that deductive mathematical methods are admissible as one among several methodological tools one can use in phenomenology. As Husserl says in *Phenomenological Psychology*, "The mediate, concluding, and deducing procedure is not lacking at higher stages [of phenomenology]… but by no means is the entire science of the type of a mathematics" (*PP*, p. 37). Axioms and deductions and the (in some cases, definite) manifolds they govern are legitimate as *part of* phenomenology, but there will always be more to phenomenology than what can be secured by deductive inference. Phenomenology as a whole is an open-ended, ever-expanding field of research whose pursuit is, as Husserl famously put it, an "infinite task" (*Crisis*, p. 291).

Phenomenology, then, is not mathematical. It does not rely on deductive inference, it does not traffic in exact objects, it is not satisfied by a "definite manifold" of possible objects. But mathematics, with all its deductive rules of inference and exact concepts, *can* be applied to the study of inexact conscious experiences, just as mathematics can be applied to the study of inexact physical objects or organisms or social systems (mathematical physics, mathematical biology, mathematical social science). Indeed, to formulate the kinds of complex claims Husserl does (in, e.g., his theory of the constitution of space), the cognitive aid supplied by symbolic representations is essential. And as long as these symbol systems are properly distinguished from the reality they symbolize, Husserl has no objection to their use.

Husserl and Dynamical Systems Theory

An especially well-developed area of contemporary formal theory, which is common to many natural sciences, is dynamical systems theory.[38] A dynamical system is a rule which says how a system will evolve over time. Dynamical systems are pervasive in science insofar as iterated functions and most differential equations are dynamical systems.[39] Dynamical systems theory is commonly used in neuroscience and cognitive science, and has also been of considerable interest in recent philosophy of cognitive science, insofar as it is taken to provide a novel explanatory framework.[40]

The relevance of dynamical systems theory to the study of consciousness should be immediately apparent, insofar as conscious processes unfold over time in a structured way. Indeed, several authors have looked at Husserlian themes from a dynamical systems perspective (see note 2). What follows can be seen a providing a foundational analysis relative to these studies, an analysis of the basic assumptions of dynamical system theory insofar as they appear in Husserlian phenomenology.

Mathematically, a dynamical system is a map $\phi : S \times T \rightarrow S$, which associates an "initial condition" s in a "state space" S with a future time t in the "time space" T.[41] A dynamical system determines a set of sequences of states ("orbits" or, for continuous dynamical systems, "trajectories") in a state-space. You plug in an initial state and a future time and the map ϕ tells you what state the system will have evolved to at that

[38] For a visual and intuitive overview of dynamical systems theory see Abraham and Shaw (1982). For a more formal treatment see Perko (1996).
[39] A differential equation is a dynamical system if it satisfies the existence and uniqueness theorem for differential equations.
[40] See Beer (2000).
[41] This map must meet two conditions in order to qualify as a dynamical system: (1) for all states s in S, $\phi(s, 0) = s$, and (2) for states s in S, $\phi(s, t_1 + t_2) = \phi(\phi(s, t_1), t_2)$.

future time. Thus, dynamical systems are, by definition, deterministic, though they can manifest complex and unpredictable behavior ("chaotic" dynamics). One benefit of dynamical systems theory is that it allows the behavior of a system to be studied using topological and geometric tools—the behavior of a system can be visualized by mapping its possibilities into a space and plotting its possible behaviors in that space.

At least five fundamental assumptions of dynamical systems theory appear in Husserlian phenomenology. [42]

First, basic to dynamical systems theory is the notion that systems can be treated in terms of their possibilities. These possibilities are studied via a *state space*, (*S* in the map defined above), that is, a structured set of possibilities for a system (in what sense such a set is a *space*, and in what sense it is structured, are discussed below). Husserl applies this method to phenomenology throughout his corpus, routinely considering phenomenological problems from the standpoint of a "manifold" of possible experience. Consider the following, chronological sequence of examples. In the *Investigations* (1900) Husserl refers to "an ideally delimited manifold of possible intuitions" (*LI*, p. 692), and "a continuous series of percepts, all belonging to the perceptual manifold of one and the same object" (*LI*, p. 701). In *Ideas* 1 (1913) Husserl characterizes the world of natural science as "the sum-total of objects of possible experience and experiential cognition" (*Ideas*, p. 46, Kersten translation) and says that an individual object is represented by a "system of all possible 'subjective modes of appearing'" (*Ideas*, p. 346). In the *Analyses Concerning Passive and Active Synthesis* (lectures first given in 1921) Husserl refers to

[42] One important caveat to this discussion is that dynamical systems are essentially *closed*, in that in order for the map ϕ to be well defined we cannot allow external, unpredictable sources of input in to the system. But of course the mind and brain are open, in the sense that they are subject to forces from the external environment. In ongoing work with a mathematician (Scott Hotton) I am analyzing such systems with the explicit goal of providing a framework for formalizing dynamical claims in phenomenology and neuroscience.

"every possible object of every possible consciousness," (*APS*, p. 57). In the *Cartesian Meditations* (1929), Husserl describes phenomenology as a theory of the "actuality and potentiality of intentional life" (*CM*, p. 44). In *The Crisis of European Sciences and Transcendental Phenomenology* (1934-1937) Husserl describes the "lifeworld," as a "universe of what is intuitable in principle" (*Crisis*, p. 127), that is, a universe of things, each of which "indicates an ideal general set of actual and possible experiential manners of givenness" (*Crisis*, p. 166).

Second, the state space of a dynamical system can typically be parameterized by a set of state-variables $x_1, x_2, ,..x_n$ which correspond to the *n* degrees of freedom of the system. Husserl also parameterizes his manifolds of possible experience and thereby treats them as multidimensional. In *Thing and Space*, for example, Husserl uses one variable to characterize felt orientation of the eyes, and a set of additional variables to characterize felt orientations of different body parts ("kinesthetic sensations pertaining to the head, the trunk, etc." (*TS*, p. 168):

> It is not only the eye that might move but also the remainder of the body…we thus have a complex of; variables (*K, K′,K″*,…) which are independently variable in relation to one another but in such a way that they form a system wherein each of the variables always have a definite value (*TS*, p. 168).

Thus, one's total sense of one's body at a time can be represented by a point on an *n*-dimensional manifold, where *n* corresponds to the number of body parts (each represented by a *K*) whose orientation can be felt.

Third, thinking of a manifold of possibilities as a multi-dimensional space allows one to use variational methods to study relationships between these dimensions. Such techniques are common in dynamical systems theory and make up a central component of

Husserlian phenomenology. In *Logical Investigations*, for example, Husserl parses intentional acts into various types of constituent in order to vary these constituents and thereby identify their pairwise dependencies (see, e.g., *LI* 5, section 20). In the example from *Thing and Space* above, Husserl "freezes" the body in order to isolate the contribution of ocular movement to the constitution of space: "we thought of K', K'', … as constant and then of K as sometimes constant, sometimes arbitrarily varying" (*TS*, p. 168). In later works these techniques are developed into an explicit method: the method of free variation (where one imagines arbitrary variations on an exemplar in some class of possible experience in order to identify what remains constant throughout the variation).[43] It may be that Husserl was convinced of the power of variational methods by his early mathematical work on the calculus of variations, which culminated in his dissertation on the topic.[44]

Fourth, the state spaces of dynamical systems have topological and geometric structure. State space are typically topological manifolds which can take a variety of specific forms: curves, spheres, cylinders, tori, Möbius strips, etc. The state space of a pendulum (its possible positions and velocities), for example, is a u-shaped cylinder in a three dimensional space. The activation space of a neural network with 50 neurons (the set of possible patterns of firing of the 50 neurons) is a 50-dimensional hypercube. In the latter case it is typical to introduce a Euclidean metric as well, to study how network states cluster into similarity classes. The sets Husserl posits are also plausibly construed

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[43] For recent discussion of the method of free variation, with reference to earlier works, see Tieszen (2005).

[44] The calculus of variations is a technique for finding paths that minimize or maximize a function defined on a set of curves. In effect, the mathematician considers all possible curves connecting two points in an abstract space to find, for example, a path of least action across a surface. There are notable differences between free variation in phenomenology and the calculus variations. Phenomenological free variation does not involve any explicit numerical calculation, and neither does it issue in a single solution to some equation (in the calculus of variations one solves a differential equation to identify a single path which maximizes or minimizes a function). Nonetheless, both techniques involve variation in a space of possibilities to secure mathematically rigorous results.

as manifolds with topological and geometric structure. For example, Husserl describes experienced space as a conjunction of two manifolds—a “linear manifold of receding,” and a “cyclical manifold of turning” (*TS*, p. 216).[45] We have also seen that Husserl thought of some manifolds of possibility as having metrical structure, insofar as he takes “distances” between perceptual acts to be meaningful (*LI* 6, section 24).

Fifth, dynamical rules determine the order in which the possibilities of a system are visited. Based on the physical properties of a pendulum, only certain behaviors will be observed. If there were no such rules, then any arbitrary sequence of pendulum states could occur; for example the pendulum might flicker from one position to another without any spatial continuity. Similar analyses are in Husserl. Indeed, an overarching theme of transcendental phenomenology is that, for experiential processes to present one with an external, “transcendent” world enduring through time, the subject’s own experience over time must obey certain laws. Streams of experience, which can be parsed into sub-streams instantiating elements of the various manifolds Husserl describes—our bodily experiences over time, our visual experience over time, etc.—must proceed in a lawful way in order for objective reality to appear at all. Not just any random sequence of possible experiences is sufficient to present a world. If there were no correlation between how we move and what we see, then we would have a radically diminished or even “annihilated” sense of the world. If current experience were not transformed into past experience in a law-governed way, we would have no stable sense of present or past or indeed of time as we know it. Husserl makes these general points in a striking passage at

[45] Both kinds of manifold are treated of explicitly in the manuscript “sets and manifolds” of 1893. (See *Aufsätze und Rezensionen*, section II, Number 1, “On Sets and Manifolds”).

the end of *Thing and Space*, where he admits the possibility that such laws could fail to hold:

> Could it not be that, from one temporal moment on, all harmonious fulfillment would cease and the series of appearances would run into one another in such a way that no posited unity could ultimately be maintained…Could it not happen that… the entire stream of appearance dissolve into a mere tumult of meaningless sensations. Thus we arrive at the possibility of a phenomenological maelstrom … it would be a maelstrom so meaningless that there would be no I and no thou, as well as no physical world—in short, no reality (*TS*, pp. 249-250).[46]

The task of transcendental phenomenology is precisely to show how it is that experience is *not* a maelstrom, to show how it is that, insofar as a transcendent world appears at all, elements of sets of possible experience must be instantiated in accordance with mathematically describable rules.

One might ask: how does this discussion of dynamical systems and phenomenology relate to other areas in which Husserl and others have provided some formalization, such as his theory of time consciousness, perception, horizon theory, and mereology? I think a detailed story can be told in each case. I will focus here on mereology, as an instructive exemplar in the kind of interaction I envision between dynamical systems theory and other areas of formal phenomenology

Biology provides a useful parallel case, insofar as it studies entities which are both mereologically and dynamically complex. Organisms are parts of higher-level aggregates like ecosystems, and can be decomposed into organs, tissues, cells, organelles, macromolecules, molecules, atoms, and sub-atomic particles. Dynamical systems

[46] Husserl makes the point more famously in *Ideas*, where he describes the possible annihilation of the world as involving a disruption in the orderly stream of experience, whereby it would "lose its fixed regular organizations of adumbrations, apprehensions, and appearances" (*Ideas*, p. 109; Kersten translation).

analyses exist at every level of this mereological hierarchy. A famous dynamical system, the Lotka-Volterra predator-prey model, describes fluctuations in populations of predators and prey. Cellular networks, in particular neural networks, are routinely treated as dynamical systems—a prominent example is Hopfield's model of recurrent neural networks as memory systems, in which the stable fixed points of a network correspond to its memories. The well known Hodgkin-Huxley equations, describe the voltage potential of individual cells (neurons) as a function of ion concentrations. Within individual cells chemical signaling is frequently studied using dynamical equations, and even at the lowest level (or something near it), subatomic particles like electrons are modeled by quantum electrodynamics, which describes the probability density functions for electron position as a function of time. The point of all of this is to show that mereology and dynamical systems theory are complementary ways of studying a complex system. Mereology shows how a given system breaks down into smaller systems (or aggregates into larger ones), and at each level of organization dynamical systems theory shows how the system, at that scale, evolves over time.

Mereology and dynamical systems theory interact in a similar way in phenomenology. One can describe the rules which must be obeyed in order for unfolding total experiences to be coherent at all, or one can abstract visual experience as a mereological part from the total stream and then ask what rules must be obeyed in order for specifically visual experience to be coherent. One could further focus on a single object in the visual field and ask what rules have to be obeyed in order for that object to appear coherently. Thus, in phenomenology, as in other sciences, distinct rules of evolution exist at each level of a mereological hierarchy.

Conclusion

In the *Phenomenological Psychology*, Husserl says, "We proceed 'scientifically,' but we wish to know nothing of 'scientificality'" (*PP*, p. 172). That is, phenomenology endeavors to draw on tools and methods of science without committing the errors of an unbridled "scientificality," without, for example, confusing the mathematics which describes the world with the world itself; or confusing the calculation procedures of deductive logic with actual thought processes. If one so proceeds, then one can safely make use of mathematics in phenomenological settings.

I have argued that a mathematical discipline especially resonant with Husserl's own, tentative formalization is dynamical systems theory. I believe that a dynamical approach to Husserlian phenomenology holds out considerable promise for integrating phenomenology and cognitive science. In particular, if, as I have argued elsewhere, one can define a function which associates possible brain states with possible conscious states, then relations between the dynamics of the brain as described by computational neuroscience and the dynamics of consciousness as described by Husserl can be pursued with mathematical precision. Drawing the parallels properly will require a suitably mathematized phenomenology.[47]

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References

Abraham, Ralph and Shaw, C. 1982. *Dynamics—The Geometry of Behavior*. (4 Volumes). Santa Cruz, CA: Aerial Press.

Barus, Imants. 1989, Categorical Modeling of Husserl's Intentionality. *Husserl Studies* 6: 25-41. TODO.

Beer, R. 2000. Dynamical Approaches to Cognitive Science. *Trends in Cognitive Science*, 4: 91-99.

Blecksmith, R. and Null, G. 1991. Matrix Representation of Husserl's Part-Whole-Foundation Theory. *Notre Dame Journal of Formal Logic*, 32: 87-111.

Chokr, N. 1992. Mind, consciousness, and cognition: Phenomenology vs. cognitive science, *Husserl Studies* 9: 179-197.

Da Silva, J.J. 2000. Husserl's Two Notions of Completeness. *Synthese* 125: 417-438.

Dreyfus, H. (ed). 1984. *Husserl, Intentionality, and Cognitive Science*. Cambridge, MA: MIT Press.

Gärdenfors, P. 2000. *Conceptual Spaces: The Geometry of Thought*. Cambridge, MA: MIT Press.

Fine, K. 1995. Parts and Wholes. In: Smith and Smith (eds), *The Cambridge Companion to Husserl.* Cambridge: Cambridge University Press.

Hill, C.O. and Haddock, G.E.R. (eds). 2000. *Husserl or Frege? Meaning, objectivity, and mathematics*. Chicago, IL: Open Court.

Hill, C.O. 1997. Did Georg Cantor Influence Edmund Husserl? *Synthese* 113: 145-170.

Kasmier, D. 2003. *Husserl's Theory of A Priori Knowledge*. Doctoral dissertation: University of Southern California.

Miller, I. 1984. *Husserl, perception, and temporal awareness*. Cambridge, MA: MIT Press.

Mohanty, J. 1999. *Logic, truth, and the modalities: from a phenomenological perspective.* Dordrecht: Springer.

Husserl, E. [1886-1901] 1983. *Studien zur Arithmetik und Geometrie*. I. Strohmeyer (ed). Dordrecht: Springer.

Husserl, E. [1890-1910] 1979. *Aufsätze und Rezensionen 1890-1910*. B. Rang (ed). Dordrecht: Springer.

Husserl, E. [1891]. 2003 Philosophy of Arithmetic (*PA*), trans. D. Willard. Dordrecht: Springer.

Husserl, E. [1900] 1973. *Logical Investigations (LI)*, trans. J.N. Findlay. London: Routledge and Kegan Paul.

Husserl, E. [1905-1920] 1991. *On the Phenomenology of the Consciousness of Internal Time (Time)*, trans. J. Brough. Dordrecht: Springer.

Husserl, E. [1907] 1997. *Thing and Space: Lectures of 1907 (TS)*, trans. R. Rojcewicz. Dordrecht: Springer.

Husserl, E. [1913] 1962. *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy. First Book: General Introduction to a Pure Phenomenology* (*Ideas*), trans. R.B. Gibson. New York: Collier. Page numbers are from this translation unless otherwise noted.

Husserl, E. [1913] 1980. *Ideas Pertaining to a Pure Phenomenology and to a*

Phenomenological Philosophy. First Book: General Introduction to a Pure Phenomenology (*Ideas*), trans. F. Kersten. Dordrecht: Springer.

Husserl, E. [1913] 1980. *Ideas Pertaining to a Pure Phenomenlogy and to a Phenomenological Philosophy. Third Book: Phenomenology and the Foundations of Science* (*I3*), trans. T.E. Klein and W.E. Pohl. Dordrecht: Springer.

Husserl, E. [1917-1918] 2001. *Die `Bernauer Manuskripte' über das Zeitbewußtsein (1917/18)*. Bernet, R. and Lohmar, D. (eds). Dordrecht: Springer.

Husserl, E. [1920-26] 2001. *Analyses of Passive and Active Synthesis* (*APS*), trans. A. Steinbock. Dordrecht: Springer.

Husserl, E. [1925] 1975. *Phenomenological Psychology. Lectures, Summer Semester 1925 (PP)*, trans. J. Scanlon. The Hauge: Nijhoff.

Husserl, E. [1927-1931] 1997. *Psychological and Transcendental Phenomenology and the Confrontation with Heidegger 1927-1931*, trans. T. Sheehan and R.E. Palmer. Dordrecht: Springer.

Husserl, E. [1929] 1960. *Cartesian Meditations (CM)*, trans. Dorion Cairns. The Hague: Nijhoff.

Husserl, E. [1929] 1969. *Formal and Transcendental Logic* (*FTL*), trans. D. Cairns. The Hague: Nijhoff.

Husserl, E. [1934-1937] 1970. *The Crisis of European Sciences and Transcendental Phenomenology: Introduction to Phenomenological Philosophy* (*Crisis*), trans. D. Carr. Evanston IL: Northwestern University Press.

Husserl, E. [1939] 1973. *Experience and Judgment (EJ)*, trans. J.S. Churchill and K. Ameriks. Evanston, IL: Northwestern University Press, 1973.

Husserl, E. 1981. *Shorter Works* (*SW*). P. McCormick and F. Elliston (eds). Notre Dame: University of Notre Dame Press.

Krysztofiak, W. 1995. Noemata and Their Formalization. *Synthese* 105: 53-86.

Majer , U. 1997. Husserl and Hilbert on Completeness. *Synthese* 110: 37-56.

Mensch, J.R. 1991. Phenomenology and artificial intelligence: Husserl learns Chinese. *Husserl Studies* 8:107-127.

Narens, L. 2002. *Theories of Meaningfulness*. Mahwah, NJ: Lawrence Erlbaum Associates.

Mancosu, P. and Ryckman, T.A. 2002. Mathematics and Phenomenology: The Correspondence between O. Becker and H. Weyl. *Philosophia Mathematica* 10: 102-129.

McIntyhre, R. 1986. Husserl and the Representational Theory of Mind. *Topoi* 5:101-113.

Miller, I. 1984. *Husserl, Perception, and Temporal Awareness*. Cambridge, MA: MIT Press.

Perko, Lawrence 1996. *Differential Equations and Dynamical Systems* (Second Edition). New York: Springer.

Petitot, J. 1999. Morphological Eidetics for a Phenomenology of Perception. In: Francisco Varela, et al. eds. *Naturalizing Phenomenology*. Stanford, CA: Stanford University Press.

Petitot, J., Varela F., Pachoud B., Roy J-M. (eds). 1999. *Naturalizing Phenomenology*. Stanford, CA: Stanford University Press.

Sharoff, S. 1995. Phenomenology and cognitive science. In *Stanford Humanities Review*

4: 189-204.
Simons, P. 1987. *Parts: A Study in Ontology*. Oxford: Oxford University Press.
Smith, B. (ed). 1982. *Parts and Moments: Studies in Logic and Formal Ontology*. Munich: Philosophia Verlag.
Smith, B. (ed). 1998. The Basic Tools of Formal Ontology. In N. Guarino (ed), *Formal Ontology in Information Systems*. Amsterdam: IOS Press
Smith, D.W. and Smith, B, (eds). 1995. *The Cambridge Companion to Husserl*. Cambridge: Cambridge University Press.
Smith, D.W. 1995. Mind and Body. In: D.W. Smith and B. Smith (eds), *The Cambridge Companion to Husserl*. Cambridge: Cambridge University Press.
Smith, D.W. 2002. Mathematical Form in the World. *Philosophia Mathematica* 10: 102-129.
Smith, D.W. and McIntyre, R. 1982. *Husserl and Intentionality*. Dordrecht: Reidel Publishing Company.
Tieszen, R. 1989. *Mathematical Intuition: Phenomenology and Mathematical Knowledge*. Dordrecht: Kluwer.
Tieszen, R. 2005. *Phenomenology, Logic, and the Philosophy of Mathematics*. Cambridge, MI: Cambridge University Press.
Tragesser, R. 1977. *Phenomenology and Logic*. Ithaca, NY: Cornell University Press.
Tragesser, R. 1984. *Husserl and Realism in Logic and Mathematics*. Cambridge: Cambridge University Press.
Van Gelder T. 1996. Wooden Iron? Husserlian Phenomenology Meets Cognitive Science. *Electronic Journal of Analytic Philosophy* 4.
Welton, D. 2000. *The Other Husserl: The horizons of transcendental phenomenology*. Bloomington, IA: Indiana University Press.
Willard, D. 1979. Husserl's Critique of 'Extensional' Logic: A Logic That Does Not Understand Itself. *Idealistic Studies* IX: 142-164.
Willard, D. 1980. Husserl on a Logic that Failed. *The Philosophical Review* 89: 46-64.
Willard, D. 1984. *Logic and the objectivity of knowledge : a study in Husserl's early philosophy* Athens, Ohio: Ohio University Press.
Yoshimi, J. 2001. *Dynamics of Consciousness: Neuroscience, Phenomenology, and Dynamical Systems Theory*. Doctoral dissertation: UC Irvine.